

Quartic Gauge Boson Couplings

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Quartic vertices provide a window into one of the most important problems in particle physics; the understanding of electroweak symmetry breaking. I survey the various processes that have been proposed to study quartic gauge boson couplings at future e^+e^- , $e\gamma$, $\gamma\gamma$, e^-e^- , and pp colliders. For the lowest dimension operators that do not include photons, it appears that the LHC will provide the most constraining measurements. However, precision measurements at high energy e^+e^- colliders involving W^+W^- rescattering are also quite sensitive to the effects of a strongly interacting weak interaction. For quartic couplings involving photons, $\gamma\gamma$ collisions appear to be the best place to measure these couplings. Measurements using gauge boson production in $e\gamma$ collisions are almost as precise as the $\gamma\gamma$ processes with $e^+e^- \rightarrow VVV$ about an order or magnitude less sensitive.

I. INTRODUCTION

The non-Abelian gauge nature of the standard model predicts, in addition to the trilinear WWZ and $WW\gamma$ couplings (TGV's), quartic gauge boson couplings (QC's). The strength of the couplings is set by the universal gauge couplings of the $SU(2)$ local gauge symmetry. In the standard model there are only three quartic couplings which necessarily involve at least two charged W 's; $W^+W^-W^+W^-$, W^+W^-ZZ , and $W^+W^-\gamma\gamma$. Although the $ZZZZ$ vertex is not present in the SM it is present at tree level via Higgs exchange while the $\gamma\gamma ZZ$ vertex is only produced at loop level in the Standard Model.

The trilinear and quartic couplings probe different aspects of the weak interactions. The trilinear couplings test the non-Abelian gauge structure where deviations from the SM can result from integrating out heavy particles in loops (1). In contrast, the quartic couplings can be regarded as a window on electroweak symmetry breaking. Recall that the longitudinal components of the W and Z are Goldstone bosons. The quartic couplings of gauge bosons therefore represent a connection to the scalar sector of the theory. The QC's

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would arise as a contact interaction manifestation of heavy particle exchange.

It is quite possible that the quartic couplings deviate from their SM values while the TGV's do not. For example, the BESS model is a non-linear realization of symmetry breaking where new structures not present at tree level appear in $4W$ couplings (2). There are models with a heavy scalar singlet interacting with the Higgs sector which do not affect the ρ parameter nor the TGV's but do change the $4W$ vertex (3).

Thus, if the mechanism for electroweak symmetry breaking does not reveal itself through the discovery of new particles such as the Higgs boson, supersymmetric particles, or technipions it is quite possible that anomalous quartic couplings could be our first probes into this sector of the electroweak theory.

While considerable effort has been expended to study the trilinear couplings, the quartic couplings are only starting to receive much attention. In this contribution I review recent developments in the study of quartic couplings and attempt to summarize the current status of this subject. In the next section I describe the effective Lagrangians relevant to QC's. I will then describe various processes that have been proposed to study quartic couplings using a wide variety of colliding particles: pp , e^+e^- , $e\gamma$, $\gamma\gamma$, and e^-e^- . In the final section I summarize these results and also add some comments as to where the subject can benefit from further work.

II. EFFECTIVE LAGRANGIANS AND QUARTIC COUPLINGS

The formalism of effective Lagrangians provides a well-defined framework for investigating the physics of anomalous couplings and electroweak symmetry breaking (4–6). The infinite set of terms in \mathcal{L}_{eff} can be organized in an energy expansion where at low energy only a finite number of terms will contribute to a given process. At higher energies more and more terms become important until the whole process breaks down at the scale of new physics. One focuses on the leading operators in the expansion.

Quartic operators can either be associated with trilinear couplings or can be genuinely quartic. The former type is described by:

$$\mathcal{L}^{WW\gamma} = -ie \frac{\lambda_\gamma}{M_W^2} F^{\mu\nu} W_{\mu\alpha}^\dagger W_\nu^\alpha \quad (1)$$

This operator generates $WW\gamma\gamma$ couplings with strength $e^2\lambda_\gamma$ in addition to $WW\gamma$ couplings. These vertices are not likely to be particularly interesting as the parameter λ_γ will already be constrained from other processes such as $e^+e^- \rightarrow WW$ where the TGV contributes but the QC does not appear (1)

We will restrict our discussion to the more interesting, genuinely quartic couplings. We concentrate on the lowest dimension operators that can contribute to a given vertex. We impose custodial $SU(2)$, which is satisfied to high precision by the nearness of the ρ parameter to unity, and $U(1)_{em}$ for the operators involving photons. There are 2 (equivalent) parametrizations

which have appeared in the literature. We will begin by describing these parametrizations.

A. General Parametrization

This parametrization was introduced by Bélanger and Boudjema (7,8). There are only two dimension four operators. They do not involve photons since $U(1)_{em}$ requires derivatives which would result in a higher dimension operator. Imposing $SU(2)_C$ the two dimension four operators are given by:

$$\begin{aligned}\mathcal{L}_4^o &= \frac{1}{4}g_o g_W^2 (\vec{W}_\mu \cdot \vec{W}^\mu)^2 \\ &\rightarrow g_o g_W^2 [(W_\mu^+ W^{-\mu})(W_\nu^+ W^{-\nu}) + \frac{1}{c_w^2} W_\mu^+ W_\mu^- Z^\nu Z_\nu + \frac{1}{4c_w^2} Z^\mu Z_\mu Z_\nu Z^\nu] \\ \mathcal{L}_4^c &= \frac{1}{4}g_c g_W^2 (\vec{W}_\mu \cdot \vec{W}^\mu)(\vec{W}^\mu \cdot \vec{W}_\nu) \\ &\rightarrow g_c g_W^2 [\frac{1}{2}(W_\mu^+ W^{-\mu} W_\nu^+ W^{-\nu} + W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu}) \\ &\quad + \frac{1}{c_w^2} W_\mu^+ W^{-\nu} Z^\mu Z_\nu + \frac{1}{4c_w^2} Z^\mu Z_\mu Z_\nu Z^\nu]\end{aligned}$$

These operators involve the maximum number of longitudinal modes. These are the most important manifestations of an alternative symmetry breaking scenario. Note that the $ZZZZ$ vertex does not appear in the SM and $W^+W^-W^+W^-$ cannot be probed via 3 boson production in e^+e^- . Photons do not appear in these genuine QC's.

The first operator can be thought of as parametrizing heavy neutral scalar exchange so that we can make the connection:

$$g_o \propto \kappa^2 \left(\frac{M_W^2}{\Lambda^2} \right) \quad (2)$$

where κ is the strength of coupling in the W system. Heavy Higgs exchange, at tree level in the SM, gives κ of order 1. In this case $g_o \simeq 0.2$ corresponds to $M_H = \Lambda \sim 180$ GeV which would most likely be observed directly at a high energy collider invalidating this approach. On the other hand, taking a Higgs mass of 1 TeV yields a contact term of strength $g_o \simeq 6 \times 10^{-3}$. Thus, to see the effect of a heavy scalar as a deviation to the QC requires very precise measurements.

For the case of scalar exchange $g_o > 0$. In the second operator the $WWZZ$ vertex corresponds to heavy charged scalar exchange so that we could associate it with a triplet of heavy scalars. A specific case of interest is when $g_o = -g_c = g_s < 0$ which could parametrize heavy vector particle exchange which might arise in theories like technicolour. In this case the $4Z$ couplings cancel and the net effect is a rescaling of the SM $4W$ vertex. Bounds on g_s therefore determine the precision with which $4W$ couplings could be measured.

To introduce photons we have to go to dimension 6 operators. We only consider these dim-6 operators since they result in the largest phase space and are therefore most likely to give the largest deviations. Imposing $SU(2)_C$ and $U(1)_{QED}$ and restricting the phenomenological analysis to the C and P conserving operators with a maximum of two photons involves the $\gamma\gamma W^+W^-$ and $\gamma\gamma ZZ$ vertices described by the operators:

$$\mathcal{L}_6^0 = -\frac{\pi\alpha}{4\Lambda^2} a_0 F_{\alpha\beta} F^{\alpha\beta} (\vec{W}_\mu \cdot \vec{W}^\mu) \quad (3)$$

$$\mathcal{L}_6^c = -\frac{\pi\alpha}{4\Lambda^2} a_c F_{\alpha\mu} F^{\alpha\nu} (\vec{W}_\mu \cdot \vec{W}^\nu) \quad (4)$$

Where \vec{W}_μ is an $SU(2)$ triplet and $F^{\mu\nu}$ and $\vec{W}^{\mu\nu}$ are the $U(1)_{em}$ and $SU(2)$ field strengths respectively. Both operators have contributions from loops but the first can originate from heavy neutral scalar exchange while the second can arise from charged scalars. Note that the $SU(2)$ gauge symmetry predicts that $\gamma\gamma ZZ$ does not appear in the SM. The custodial symmetry imposed on these couplings means that, in leading order in s , they contribute in the same way to the $\gamma\gamma \rightarrow WW$ and to $\gamma\gamma \rightarrow ZZ$.

There is an additional operator which gives a $W^+W^-Z\gamma$ vertex (9):

$$\mathcal{L}^n = \frac{i\pi\alpha}{4\Lambda^2} a_n \varepsilon_{ijk} W_{\mu\alpha}^i W_\nu^j W^{k\alpha} F^{\mu\nu} \quad (5)$$

The parameter Λ is an unknown “new physics” scale which is often taken to be M_W . This is a little misleading as Λ represents the scale of new physics which one might expect to be $\mathcal{O}(1)$ TeV. One should keep this in mind when gauging the sensitivity of various experiments to the parameters a_i . To facilitate the comparison of different processes I have taken $\Lambda = 1$ TeV, rescaling results where necessary.

\mathcal{L}_0 and \mathcal{L}_c affect the value of Δr and therefore contribute to the S and T parameters (10) leading to the rather weak one-sigma constraints (9):

$$\begin{aligned} -700 &< a_0 < 100 \\ -1700 &< a_c < 900. \end{aligned} \quad (6)$$

There are no similar low energy constraints on a_n .

B. Non-Linear Realization

Another widely used effective Lagrangian is the Chiral Lagrangian. It assumes a heavy Higgs boson using a non-linear realization of the Goldstone bosons and assumes a custodial $SU(2)$ (5,11,12).

$$\mathcal{L}_1 = \frac{L_1}{16\pi^2} [Tr(D^\mu \Sigma^\dagger D_\mu \Sigma)]^2 \quad (7)$$

$$\mathcal{L}_2 = \frac{L_2}{16\pi^2} [Tr(D^\mu \Sigma^\dagger D_\nu \Sigma)]^2 \quad (8)$$

where $\Sigma = \exp(iw^i\tau^i/v)$, $v = 246$ GeV, and $D_\mu\Sigma = \partial_\mu\Sigma + \frac{1}{2}igW_\mu^i\tau^i\Sigma - \frac{1}{2}ig'B_\mu\Sigma\tau^3$. In this approach $\mathcal{L}_{1,2}$ would be the most important manifestation of alternative symmetry breaking scenarios in a Higgsless world.

The two approaches are not distinct so that $\mathcal{L}_{1,2}$ is equivalent to $\mathcal{L}_4^{o,c}$ with the mapping:

$$g_{o,c} = \frac{e^2}{16\pi^2} \frac{1}{s_w^2} L_{1,2} \quad (9)$$

Typical models with Goldstone bosons interacting with a scalar, isoscalar resonance like the Higgs boson give $L_i \sim \mathcal{O}(1)$ (13). From precision measurements of the Z^0 widths Dawson and Valencia obtained the weak bounds $-28 \leq L_1 + \frac{3}{2}L_2 \leq 26$ (14). Imposing perturbative unitarity gives the rough constraints of $|L_1| \leq 0.3$ (15). Therefore, the genuine quartic couplings are presently not well constrained by experiment but are limited by perturbative unitarity. To facilitate comparison of different processes I have presented results in terms of $L_{1,2}$, rescaling results where necessary (using $\alpha = 1/128$ and $\sin_w^2 = 0.23$). I have defined L_s when $L_1 = -L_2$.

Similarly, one can write down operators involving two photons in the Chiral Lagrangian (6):

$$\mathcal{L}_o^{2\gamma} = -\frac{L_o^{2\gamma}}{\Lambda^2} \{K_o^W g^2 \text{Tr}(W_{\mu\nu}W^{\mu\nu}) + K_o^B g'^2 \text{Tr}(B_{\mu\nu}B^{\mu\nu}) \quad (10)$$

$$+ K_o^{WB} gg' \text{Tr}(W_{\mu\nu}B^{\mu\nu})\} \text{Tr}(D^\alpha \Sigma^\dagger D_\alpha \Sigma) \quad (11)$$

$$\mathcal{L}_c^{2\gamma} = -\frac{L_c^{2\gamma}}{\Lambda^2} \{K_c^W g^2 \text{Tr}(W_{\mu\alpha}W^{\mu\beta}) + K_c^B g'^2 \text{Tr}(B_{\mu\alpha}B^{\mu\beta}) \quad (12)$$

$$+ K_o^{WB} gg' \text{Tr}(W_{\mu\alpha}B^{\mu\beta})\} \text{Tr}(D^\alpha \Sigma^\dagger D_\beta \Sigma) \quad (13)$$

where $W_{\mu\nu} = \frac{\tau^i}{2}(\partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g\epsilon^{ijk}W_\mu^i W_\nu^k)$ and $B_{\mu\nu} = \frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)\tau_3$.

For $\gamma\gamma$ reactions, by making explicit the $U(1)_{QED}$ symmetry, gives the mapping:

$$a_{o,c} = \frac{4e^2}{s_w^2} L_{o,c}^{2\gamma} (K_{o,c}^W + K_{o,c}^B + K_{o,c}^{WB}) \quad (14)$$

III. MEASUREMENT OF QUARTIC COUPLINGS

In this section I survey the various processes that have been proposed to measure quartic couplings.

A. Measurement of the Dimension 4 Operators

TABLE 1. Event rates for various VVV final states in the reaction $e^+e^- \rightarrow VVV$ for $\sqrt{s} = 500$ GeV and $L=10 \text{ fb}^{-1}$. From Bélanger and Boudjema Ref. (7).

Final State	Events	Comments
WWZ	400	$M_H < 2M_W$ or $M_H > 1 \text{ TeV}$
	460	$M_H = 200 \text{ GeV}$
ZZZ	9	$M_H > 1 \text{ TeV}$
$WW\gamma$	1356	$\left\{ \begin{array}{l} \theta_{\gamma beam} > 15^\circ \\ p_{T\gamma} > 20 \text{ GeV} \\ 2\gamma\text{'s separated by } 15^\circ \end{array} \right.$
$ZZ\gamma$	147	
$Z\gamma\gamma$	465	

1. The Processes $e^+e^- \rightarrow W^+W^-Z, ZZZ$

At an 500 GeV e^+e^- collider the W fusion process will be ineffective so that three gauge boson production may be a reasonable substitute for the measurement of quartic couplings. In the process $e^+e^- \rightarrow VVV$ four W quartic couplings don't contribute so that vertices with at least two neutral vector bosons where one of the neutrals couples to the e^+e^- vertex are likely to be the best tested in e^+e^- collisions. For any model with $SU(2)$, however, $WWWW$ vertices are related to $WWZZ$. $SU(2)$ also predicts a $4Z$ vertex which will contribute to $e^+e^- \rightarrow ZZZ$. The event rates for reactions that meet this criteria are shown in Table 1 for $\sqrt{s} = 500$ GeV and assuming an integrated luminosity of $L=10 \text{ fb}^{-1}$. The approach used is to look for deviations in the cross sections from their standard model values (7,16).

The process $e^+e^- \rightarrow W^+W^-Z$ involves TGV's, QC's and Higgs exchange. In the standard model there is a subtle cancellation between the various contributions. Only anomalous QC's were considered under the assumption that TGV's can be measured better elsewhere and assume the large M_H limit so that Higgs exchange can be neglected. The standard model cross section is 39.88 fb. In their analysis Bélanger and Boudjema included the 67% BR corresponding to the 6 jet and 4 $jet + e^\pm$ or μ^\pm final states, not including τ 's. The signal can be enhanced by using right handed electrons. The cross section is shown in Fig. 1 as a function of L_1 . The most dramatic effects are for longitudinal W 's with virtually no sensitivity in the TTU mode. The limits obtained on the couplings are based on a 3σ deviation in the total unpolarized cross-section including the 67% BR defined above and only taking statistical errors into account. One obtains the sensitivities (7):

$$\begin{aligned}
-96 &< L_1 < 81 \\
-120 &< L_2 < 120 \\
-81 &< L_s < 70
\end{aligned} \tag{15}$$

One could use distributions to distinguish between g_o and g_c . It turns out the the E_Z distribution is especially good at this.

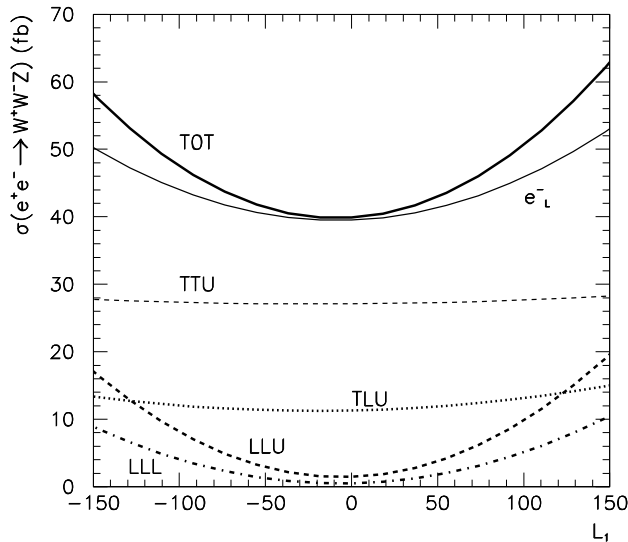


FIG. 1. Cross-section for $e^+e^- \rightarrow W^+W^-Z$ as a function of L_1 for $\sqrt{s} = 500$ GeV. Shown are the total unpolarized (TOT) cross-section, with left-handed electrons e_L^- and unpolarized cross-sections for various combinations of vector boson polarizations: T for transverse, L for longitudinal and U for unpolarized. The third label is for the Z polarization. From Bélanger and Boudjema, Ref. (7).

For the process $e^+e^- \rightarrow ZZZ$ the only SM contribution is via the Higgs boson so that the SM cross section is very small, $\simeq 1$ fb, making it very sensitive to anomalous couplings. Here Bélanger and Boudjema consider 6 jet and $4\text{ jet} + \cancel{E}$ (not including τ final states) corresponding to 87% of events (7). To use these modes one will need good invariant mass reconstruction to distinguish from the WWZ final states. The largest deviations are seen in the LLU channels. Because the event rate is so small they impose the need for 50 ZZZ events which gives the bounds:

$$-78 < L_1, L_2 < 85. \quad (16)$$

Using a less conservative, naive, 4σ deviation from the SM corresponding to 12 events gives

$$-44 < L_1, L_2 < 48. \quad (17)$$

If deviations were observed, comparing the deviations in the ZZZ mode to those found in the WWZ mode could be used to find the nature of the QC's.

2. The Processes $e^-e^- \rightarrow VV'ff'$

$e^-e^- \rightarrow VV'ff'$ is another option that has been examined (17–19). It has the advantage of no hadronic background and a low SM cross section due to

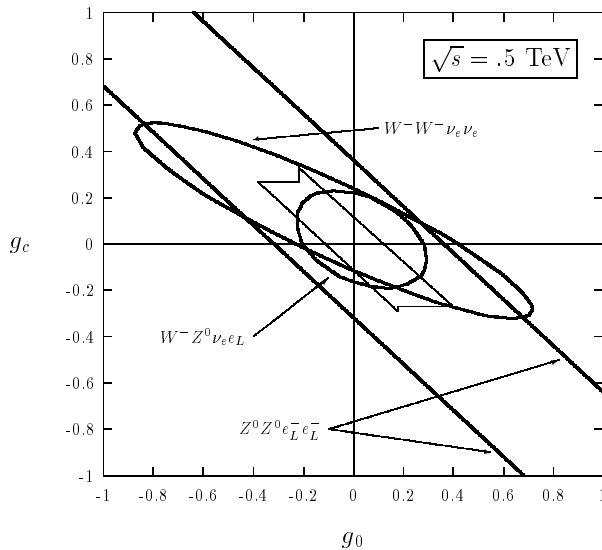


FIG. 2. Contours of observability at 95% C.L. of anomalous QC's g_o and g_c . The measurements are for $\sqrt{s} = 500$ GeV and $L=10 \text{ fb}^{-1}$. The limits which can be obtained under similar conditions in the e^+e^- mode of the same collider are indicated by the thin line. From Cuyper and Kolodziej Ref. (17).

the cancellation of diagrams. The reactions considered are:

$$\begin{aligned}
 e^-e^- &\rightarrow e^-e^-Z^0Z^0 \\
 &\rightarrow e^-\nu_e Z^0W^- \\
 &\rightarrow \nu_e\nu_e W^-W^-
 \end{aligned}
 \tag{18}$$

Note that in the last reaction only the combination $g_o + g_c$ ($L_1 + L_2$) can be probed. Cuyper and Kolodziej (17) performed an analysis assuming an integrated luminosity of 10 fb^{-1} and included a 1% systematic error. They used a 10° cut on the primary electrons and included reconstruction efficiencies. They obtained the 95% C.L. contours shown in Fig. 2.

3. *W Fusion in pp Collisions*

Although WW scattering (5,13,21) is covered by other contributions to these proceedings (22), it is a sufficiently important topic that a few brief comments are included for completeness. If the Goldstone bosons are non-linearly realized then one would expect new strong interactions at $\sim 1 \text{ TeV}$ responsible for EWSB. This might manifest itself as:

- Longitudinal W states in, for example, technicolour.

- Strong WW interactions in, for example, composite scalars.

Although $W_L W_L$ can be studied in both e^+e^- and pp colliders, because adequate W_L luminosity requires the highest energy possible it is best studied at the higher energy hadron colliders. Isoscalar resonances could be studied in W^+W^- and ZZ scattering, isovector resonances in WZ scattering and non-resonant effects in W^+W^+ . The best channel to look for the effect of genuine quartic couplings is the like-sign W pair production, $W^\pm W^\pm$. Bagger *et al.* (5) find that $pp \rightarrow W_L^+ W_L^+$ scattering at the LHC would be sensitive to $|L_1, L_2| > 1$.

4. $W_L W_L$ Rescattering in e^+e^- and $\gamma\gamma$

In $e^+e^- \rightarrow WW$ quartic couplings are studied via the effects of final state interactions (20,21). The rescattering can take place via scalar ($[I, J] = [0, 0]$) Higgs like) or vector ($[I, J] = [1, 1]$ ρ like) exchange. The W_L 's can be related to π 's via low energy theorems and chiral perturbation theory. Resonance effects for a ρ like resonance are noticeable at a $\sqrt{s} = 500$ GeV collider up to 5 TeV (20). Resonances in the $I = 2$ channel could be studied in e^-e^- .

It may also be possible to study WW rescattering at TeV energies in $\gamma\gamma$ colliders (23,24). Berger and Chanowitz (24) have examined rescattering effects in $\gamma\gamma \rightarrow ZZ$ in analogy to $\gamma\gamma \rightarrow \pi^0\pi^0$. They concluded that the background overwhelms the signal unless there are strong resonance effects from, for example, an f_{2TC} with mass ~ 3.4 TeV ($N_{TC} = 3$). A very high energy collider of $\sqrt{s_{\gamma\gamma}} = 3.2$ TeV ($\sqrt{s_{e^+e^-}} = 4$ TeV) with high luminosity, of order 100 fb^{-1} , would be needed to see its effects.

B. Measurement of Dimension 6 operators

1. The Processes $e^+e^- \rightarrow W^+W^-\gamma, ZZ\gamma, Z\gamma\gamma$

The process $e^+e^- \rightarrow W^+W^-\gamma$ is used to study the $W^+W^-\gamma\gamma$ and $W^+W^-Z\gamma$ couplings. It has the largest cross section of all 3-boson production and is quite sensitive to dimension 6 operators. The largest deviations occur when both W 's are longitudinal. Bélanger and Boudjema (7) impose the cuts $P_{T\gamma} > 20$ GeV, $\theta_{e\gamma} > 15^\circ$, and $|\eta_\gamma| < 2$ resulting in a cross section of $\sigma_{WW\gamma} = 135.6$ fb. They used the 79% of the BR that does not include τ 's with 45% being $4 \text{ jet} + \gamma$. The anomalous QC's contribute significantly to cross-sections with right handed electrons. They obtain the 3σ limits:

$$\begin{aligned} -62 &< a_o < 93 \\ -110 &< a_c < 47 \end{aligned} \tag{19}$$

The different operators give different distributions for E_γ but not for $\theta_{\gamma W}$. $e^+e^- \rightarrow ZZ\gamma$ has a SM cross-section of 14.7 fb with the same cuts as above.

It turns out that constraints obtained from $e^+e^- \rightarrow ZZ\gamma$ are less constraining than the $WW\gamma$ final state and the $e^+e^- \rightarrow Z\gamma\gamma$ cross section is very insensitive to anomalous couplings.

Leil and Stirling (25) used the process $e^+e^- \rightarrow W^+W^-\gamma$ to study a_n . They imposed the cuts $|\eta_\gamma| \leq 2$, $E_\gamma > 20\%$ to avoid collinear singularities and particle separation of 15° , obtaining a cross section for $\sqrt{s} = 500$ GeV of $\sigma_{SM} = 123.4$ fb. Using the E_γ spectrum they obtain the additional limit based on $L=10$ fb $^{-1}$ and requiring 3σ deviations of

$$-610 < a_n < 660. \quad (20)$$

2. The Processes $\gamma\gamma \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow ZZ$

These reactions are in the pure non-abelian gauge sector of the SM. Both the trilinear and quartic couplings enter. Since the TGV's can be constrained better elsewhere these reactions are ideal tests of the quartic couplings. The $WW\gamma\gamma$ and $ZZ\gamma\gamma$ couplings are related by $SU(2)$ but because they contribute to different observables we can set independent bounds on them.

The process $\gamma\gamma \rightarrow W^+W^-$ constitutes the largest cross-section in $\gamma\gamma$ collisions, with a cross-section at 400 GeV of $\sigma = 80$ pb making a $\gamma\gamma$ collider a W -Factory. The angular distributions are shown in Fig. 3. The SM contributions are peaked along the initial photon directions while the anomalous QC's are more central. Even with angular cuts the SM contributions are still large. The photon helicities can be used to separate different contributions. In the $\lambda_1 = \lambda_2$ mode ($J = 0$) the SM does not produce W 's of different helicities. This is maintained for a_o so that a_o only contributes to $J = 0$ while a_c contributes to both $J = 0$ and $J = 2$. Because a_o and a_c have the same S-wave amplitudes distinguishing them requires the use of the photon helicity amplitudes. Taking $\cos\theta < 0.7$ the SM cross-section is 17.58 pb so that statistical errors are negligible and the main source of error is systematics. For $L=10$ fb $^{-1}$ and assuming $\Delta\sigma/\sigma = 3\%$ Bélanger and Boudjema (8) obtain:

$$\begin{aligned} -7.8 < a_o < 3.1 & \quad J_Z = 0 \\ -16 < a_c < 0.56 & \quad J_Z = 0 \\ -3.1 < a_c < 3.1 & \quad J_Z = 2 \end{aligned} \quad (21)$$

Ratios could also have been used which eliminates the need to measure the $\gamma\gamma$ luminosity. Using angular distributions could give additional information.

The process $\gamma\gamma \rightarrow ZZ$ is attractive as the SM background is very small. $SU(2)$ relates the $ZZ\gamma\gamma$ vertex to the $WW\gamma\gamma$ vertex so combining this and the previous reaction is an ideal way of testing for $SU(2)$ symmetric QC's. As before, a_o contributes to $J_z = 0$ while a_c contributes to both the $J_z = 2$ and $J_z = 0$ channels making it possible to distinguish the 2 quartic couplings.

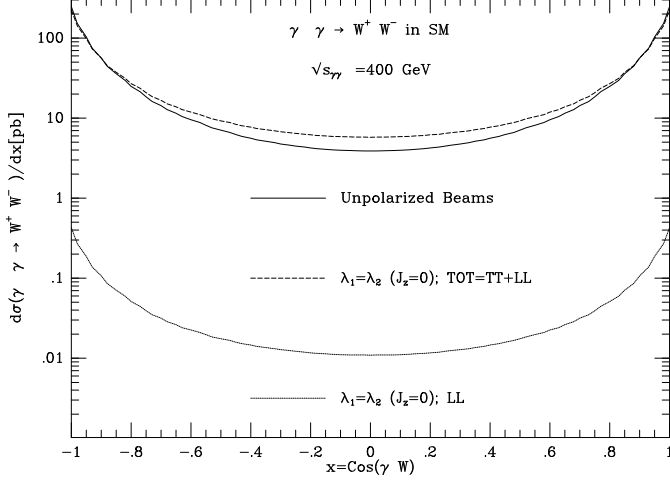


FIG. 3. W angular distribution in the process $\gamma\gamma \rightarrow W^+W^-$ at $\sqrt{s} = 400$ GeV for different initial photon helicities. From Bélanger and Boudjema, Ref. (8).

The $J_Z = 0$ and $J_Z = 2$ channels can be distinguished using polarization and angular distribution information.

Unfortunately, in the original analysis of these process it was assumed that the SM cross section was zero. A subsequent 1-loop calculation by Jikia (26) found that, although small, the SM cross section was not negligible and was dominated by the transverse modes. Nevertheless it is believed that properly including the SM contribution will still result in useful bounds, in much the same way as the $\gamma\gamma \rightarrow W^+W^-$ case (6). In the absence of a detailed analysis we describe the estimate of Baillargeon *et al.* (6). The SM $Z_L Z_L$ contribution in the heavy Higgs mass limit is quite small at all energies, ~ 1 fb. Therefore to obtain a crude estimate as to how the limits are changed it is sufficient to include the SM $Z_T Z_T$ contribution that is not affected by anomalous QC's. The limits are based on the total cross section only. One could exploit the fact that the TT cross section is relatively insensitive to the J_Z of the initial two photons to construct an asymmetry such as $\sigma(J_Z = 0) - \sigma(J_Z = 2)$ to reduce the SM background. This, of course assumes that the new physics does not contribute equally to the two J_Z . Baillargeon *et al.* include only the visible, unambiguous ZZ signal with one Z decaying hadronically and the other leptonically with the cut $\cos\theta_Z < 0.866$. The criteria of observability was based on requiring 3σ statistical deviation from the SM cross-section.

$$|a_o| < 2 \quad |a_c| < 5 \quad (\sqrt{s_{ee}} = 500 \text{ GeV } L=10 \text{ fb}^{-1})$$

TABLE 2. Sensitivities of a_o , a_c and a_n to $e\gamma \rightarrow VV'f$ corresponding to 3σ deviations, varying one coupling at a time. For events containing a photon in the final state the cut $p_{T\gamma} > 15$ GeV was used to eliminate collinear divergences. From Éboli *et al.* Ref. (9).

Final State	a_o	a_c	a_n
WWe	$-33 < a_o < 5.6$	$-230 < a_c < 220$	$-700 < a_n < 700$
$Z\gamma e$	$-150 < a_o < 150$	$-200 < a_c < 220$	—
ZZe	$-4.5 < a_o < 4.4$	$-15 < a_c < 15$	—
$W\gamma\nu$	$-87 < a_o < 84$	$-89 < a_c < 170$	—
$WZ\nu$	—	—	$-190 < a_n < 120$

$$|a_o| < 0.3 \quad |a_c| < 0.7 \quad (\sqrt{s_{ee}} = 1 \text{ TeV } L=60 \text{ fb}^{-1}) \quad (22)$$

3. The Processes $\gamma\gamma \rightarrow W^+W^-Z$ and $\gamma\gamma \rightarrow W^+W^-\gamma$

Éboli *et al.*, (27) have studied the the processes $\gamma\gamma \rightarrow W^+W^-Z$ and $\gamma\gamma \rightarrow W^+W^-\gamma$. They found that the constraints from the first reaction on a_n is as restrictive as the constraint obtained in $e\gamma$ collisions. The limits on a_c are an order of magnitude better than those coming from the e^+e^- mode and are comparable to the limits that can be obtained in the $e\gamma$ mode. However, they are a factor of 2 weaker than those obtained from $\gamma\gamma \rightarrow W^+W^-$. The limits on a_o are slightly better than those obtained in the e^+e^- mode but an order of magnitude worse than those obtained in $e\gamma$ or $\gamma\gamma \rightarrow W^+W^-$.

4. The Processes $e\gamma \rightarrow VV'f$

A number of authors have studied the effects of anomalous QC's on the reactions $e\gamma \rightarrow VV'f$ (9,15,28). The cross sections are summarized in Fig. 4 which gives the cross sections in $e\gamma$ collisions as a function of \sqrt{s} (29). The WWe and ZZe final states are most sensitive to a_o and a_c although WWe is insensitive to a_n . The cross section $\sigma(WWe)$ is an order of magnitude larger than $\sigma(WW\gamma)$ due to t-channel photon exchange. Likewise, for ZZe t-channel photon exchange is introduced by the $ZZ\gamma\gamma$ vertex, not present in the SM, making it a very sensitive process. The results from an analysis of Éboli *et al.* (9) for the sensitivities of $e\gamma \rightarrow VV'f$ to the various QC's, are summarized in Table 2. Their results are based on 3σ effects based on statistics for 10 fb^{-1} integrated luminosity. The conclusion of these studies is that they are not quite as good as those coming from $\gamma\gamma$ reactions for the study of QC's except for a_n which requires the smaller phase space process $\gamma\gamma \rightarrow W^+W^-Z^0$ in $\gamma\gamma$ collisions.

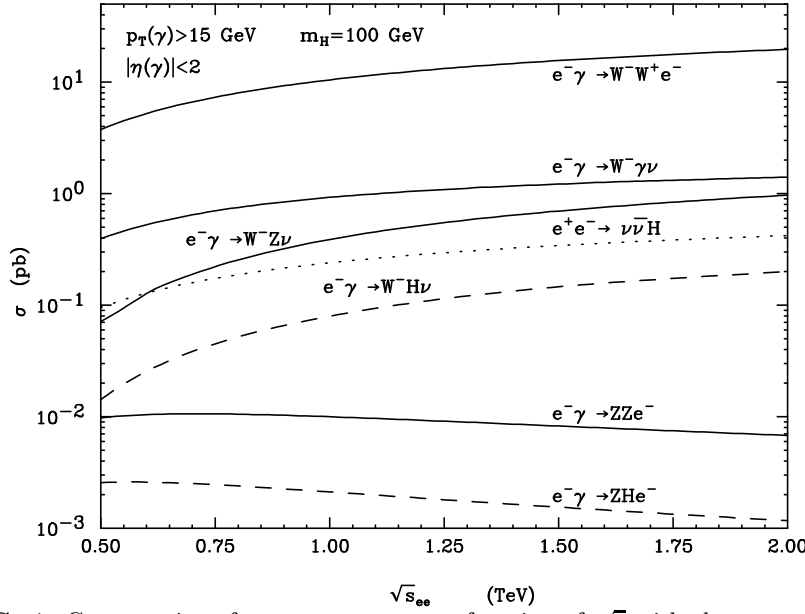


FIG. 4. Cross sections for $e\gamma$ processes as a function of \sqrt{s} with the acceptance cuts $p_T(\gamma) > 15$ GeV and $|\eta(\gamma)| < 2$. From K. Cheung Ref. (29).

IV. SUMMARY

One of the most important problems in particle physics is the understanding of electroweak symmetry breaking. If the Higgs boson is “heavy” and electroweak symmetry breaking is non-linearly realized then the quartic vertices will provide a window into EWSB. I have surveyed various processes that have been proposed to study quartic couplings. For dimension 4 operators $e^+e^- \rightarrow W^+W^-Z$, $\rightarrow ZZZ$, $e^-e^- \rightarrow VV'ff'$, $pp \rightarrow WW + X$, and WW rescattering in $e^+e^- \rightarrow W^+W^-$ have been considered and for dimension 6 operators $e^+e^- \rightarrow W^+W^- \gamma$, $ZZ\gamma$, $\gamma\gamma \rightarrow W^+W^-$, ZZ , and $e\gamma \rightarrow WW\gamma$, $ZZ\gamma$, $WZ\nu$.

For the dimension 4 operators it appears that the LHC will provide the most constraining measurements. However, is far from clear whether the LHC will be able to disentangle this sector of the weak interaction. It is possible, then, that precision measurements at high energy e^+e^- colliders through W^+W^- rescattering could be our first glimpse of a strongly interacting weak interaction.

For the dimension 6 quartic couplings involving photons, $\gamma\gamma$ collisions appear to be the best place to measure these couplings. They can be measured at least an order of magnitude more precisely than using 3-boson production in e^+e^- . Measurements using gauge boson production in $e\gamma$ collisions are almost as precise as the $\gamma\gamma$ processes.

The study of quartic couplings are still in the preliminary stages. It would be useful for the purposes of comparing different processes that a consistent parametrization of the vertices be adopted and that the different processes be analysed in a consistent way. The most dramatic effect of QC's is when all vector bosons are longitudinal. Therefore, an important next step is to include the decays of the W 's and Z 's into fermions and their reconstruction. After all, it is the fermions which are observed not the gauge bosons themselves. This would enable more sophisticated polarization studies that would simulate the experimental separation of W 's and Z 's and the separation of longitudinal and transverse gauge bosons.

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